# Construction of Circular and Rectangular Cartograms by Solving Constrained Non-linear Optimization Problems

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## 1. Introduction

A cartogram (or area cartogram) is one of the most powerful visualization tools for spatial data that has been discussed in quantitative geography (e.g., Monmonier 1977, Dorling 1996, Tobler 2004). Cartograms are transformed maps on which the areas of regions are proportional to the data values. Deformation of the shape of regions and their displacements assist map-readers to intuitively recognize the distribution of data represented on cartograms.

Cartograms are classified based on two characteristics—the shapes and contiguities of regions indicated on the cartograms. If we focus on the shapes of regions, some cartograms use complex shapes, whereas others use simple shapes such as circles and rectangles. The easiness of comparison between cartograms that have complex shapes of regions and geographical maps enables map-readers to comprehend the characteristics of spatial data presented in such cartograms. However, cartograms having complex region shapes are difficult to compare in terms of the size of the regions; in this sense, it is better to use simple shapes to express data.

Cartograms that express regions in the form of simple shapes are classified into two types based on contiguities of the regions illustrated on them.

The first type is contiguous cartograms; a rectangular cartogram proposed by Rasiz (1934) serves as an example. Rectangles represent regions, and different rectangles representing adjacent regions are placed contiguously. A rectangular cartogram is an effective visualization tool, as the size of regions is easy to perceive. However, its construction is difficult because it is impossible to maintain all the contiguities of regions in many cases and necessary to omit some of the contiguities. Therefore, although several solutions have been proposed (e.g., van Kreveld and Speckmann 2007, Speckmann et al. 2006, Heilmann et al. 2004), their applications are limited.

The second type is non-contiguous cartograms; rectangular cartograms proposed by Upton (1991) and circular (or circle) cartograms proposed by Dorling (1996) are examples. They represent regions by rectangles and circles and omit the contiguities of regions. Circular cartograms are often used for visualization because of their simple construction algorithm; however, rectangular cartograms are not used much because of their complex construction algorithm.

We think that circular and rectangular cartogram constructions are basically the same problems, and a similar approach would be quite effective. In this study, we have proposed construction solutions for both circular and rectangular cartograms by adopting an approach to the distance cartogram construction (Shimizu and Inoue 2009).

# 2. Approach to non-contiguous cartogram construction

## 2.1 Requirements for resultant cartogram shapes

We first state the requirements for non-contiguous cartogram construction. The followings are considered in the algorithm for circular cartogram construction proposed by Dorling (1996).

1) Maintain similarity between the position of circles on cartograms and the position of regions on geographical maps.
2) If possible, place the circles on top of other circles that share their border on the geographical map.
3) Avoid overlapping of circles.

Fulfillment of these requirements enhances the readability of cartograms. When people read circular cartograms, they note the differences in the size and position of circles by making comparisons with those regions on geographical maps. The large relocation of circles makes the interpretation of data difficult; therefore it is important to retain the alignments of circles in cartograms. Moreover, it is also important to avoid any overlapping of circles, as overlaps hinder the recognition of circle sizes. We agree that these requirements are essential for constructing visually elegant cartograms, and we have proposed solutions that satisfy these requirements.

## 2.2 Our approach to circular and rectangular cartogram construction

Suppose that the data to be expressed on a circular cartogram is given to every region in the target domain. Let $D_i$ denote data given to region $i$, then $r_i$, radius of the circle $i$, is given by $r_i = \sqrt{D_i/\pi}$.

Now it is possible to regard circular cartogram construction as a problem where the positions of the circles' centers must be determined. The distance between the centers of neighboring circles should be the sum of the radii of those circles, and the relative positions of the circles' centers on cartograms should resemble the corresponding regions on the geographical maps.

In fact, this description of circular cartogram construction is quite similar to that of distance cartogram construction. Distance cartograms are diagrams that visualize the proximity indices between points in a network. Its construction problem is to determine the location of points on cartograms according to the given proximity indices between points. Shimizu and Inoue (2009) formulated its construction as a non-linear optimization problem with multiple objective functions. The formulation consists of two objective functions; one minimizes the difference between the given proximity data and point distances on cartograms, and the other minimizes the difference between the direction of links, which connect the points on cartograms and geographical maps.

The difference between circular and distance cartogram constructions is that circular cartogram construction requires that circle overlapping should be avoided. Then we propose to add constraint conditions to the formulation of distance cartogram construction.

Rectangular cartogram construction is similar to circular cartogram construction; the only difference is that rectangles have width and height. Let us assume that the rectangular shapes on cartograms are decided from the shape of regions on the geographical maps, and the proportion of width per height is given. Then width $lx_i$ and height $ly_i$ of rectangle $i$ are found by $D_i$, data of region $i$. Then, the construction of rectangular cartogram is also regarded as a problem where the positions of the rectangles' centers must be determined. The distance between the centers of neighboring rectangles should be the half of the sum of the widths or heights of those rectangles, and the relative positions of the rectangles' centers on cartograms should resemble the corresponding regions on the geographical maps. This can be also formulated as a non-linear optimization problem with constraint conditions.

## 3. Formulation of non-contiguous cartogram constructions

### 3.1 Circular cartogram

Let circle $i$ on a cartogram represent region $i$ on a geographical map; ($x_i$, $y_i$) denote the x- and y-coordinates of the center of circle $i$ on the coordinate system of the cartogram, respectively; ($x^G_i$, $y^G_i$) denote the x- and y-coordinates of the centroid of circle $i$ on the geographic coordinate system, respectively; $r_i$ denote the radius of circle $i$; $d_{ij}$ denote the distance between centers of circles $i$ and $j$; $C$ denote the set of pairs of regions that share borders; $\theta^G_{ij}$ denote the bearing of the link that connects the centroids of regions $i$ and $j$ on the geographical map; and $\theta_{ij}$ denote the bearing of the link that connects centers of circles $i$ and $j$ on the cartogram.

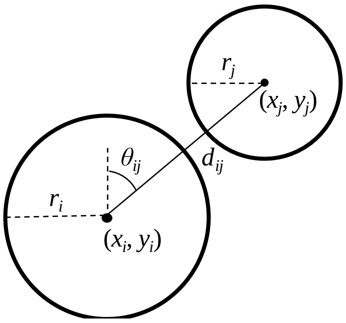


Figure 1. Neighboring circles on a circular cartogram.

The construction is formulated as Equation (1); it is composed of two objective functions and one constraint condition. The first objective function involves keeping the neighboring circles closer; the second objective function involves maintaining similarity between relative positions of circles on the cartogram and regions on the geographical map; and $\alpha$ is the weight of the objective functions. The constraint conditions between all pairs of circles are set so as to avoid overlapping of circles.

$$\underset{x,y}{Min}\left[\alpha \sum_{(i,j)\in C}\left(\frac{d_{ij}}{r_i + r_j} - 1\right)^2 + (1-\alpha)\sum_{(i,j)\in C}\left(\theta_{ij} - \theta^G_{ij}\right)^2\right]$$

$$subject\ to\ \ d_{mn} \geq r_m + r_n \quad \forall (m,n)\ \ m \neq n \tag{1}$$

$$where\ d_{ij} = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2},\ \theta_{ij} = \tan^{-1}\frac{y_i - y_j}{x_i - x_j},\ \theta^G_{ij} = \tan^{-1}\frac{y^G_i - y^G_j}{x^G_i - x^G_j},\ 0 \leq \alpha \leq 1$$

### 3.2 Rectangular cartogram

Let rectangle $i$ on a cartogram represent region $i$ on the geographical map; $lx_i$ and $ly_i$ denote the width and height of rectangle $i$, respectively; $dx_{ij}$ and $dy_{ij}$ denote the horizontal and vertical distances between centers of rectangle $i$ and $j$, respectively; and $C_x$ and $C_y$ denote the set of regions' pairs that share vertical and horizontal borders, respectively.

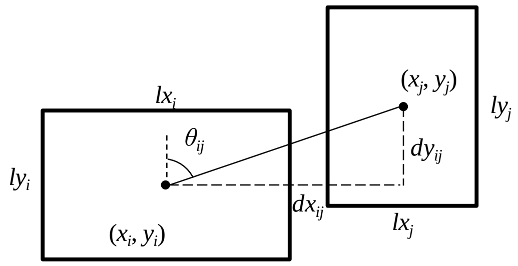


Figure 2. Neighboring rectangles that share vertical edges on a rectangular cartogram.

The rectangular cartogram construction is formulated as expressed in Equation (2), which is quite similar to Equation (1).

$$
\begin{aligned}
&\underset{x,y}{Min}\left[\alpha\sum_{(i,j)\in C_x}\left(\frac{dx_{ij}}{\frac{lx_i+lx_j}{2}}-1\right)^2+\alpha\sum_{(m,n)\in C_y}\left(\frac{dy_{mn}}{\frac{ly_m+ly_n}{2}}-1\right)^2+(1-\alpha)\sum_{(p,q)\in C_x\cup C_y}\left(\theta_{pq}-\theta_{pq}^G\right)^2\right]\\
&subject\ to\ \ dx_{st}\geq\frac{lx_s+lx_t}{2},\ dy_{st}\geq\frac{ly_s+ly_t}{2}\quad\forall(s,t)\ \ s\neq t\\
&where\ \ dx_{ij}=\left|x_i-x_j\right|,dy_{ij}=\left|y_i-y_j\right|,\ \theta_{ij}=\tan^{-1}\frac{y_i-y_j}{x_i-x_j},\ \theta_{ij}^G=\tan^{-1}\frac{y_i^G-y_j^G}{x_i^G-x_j^G},\ 0\leq\alpha\leq1
\end{aligned}
\tag{2}
$$

## 4. Application

We have applied the proposed formulation to the data from the World Population Prospects by the United Nations Population Division. We have solved the problems by a trust-region interior-point method using the mathematical optimization software *NUOPT*.

Figure 3 and 4 are the outputs that show the distribution of world population in 2010. We set that parameter $\alpha$ equals to 0.3 for the both constructions. Neighboring circles and rectangles on the cartograms are placed close to each other; there are no overlaps on the circular cartogram since all constraint conditions are satisfied for the circular cartogram construction; however, we could not get the answer that satisfies all constraint conditions for rectangular cartogram construction.

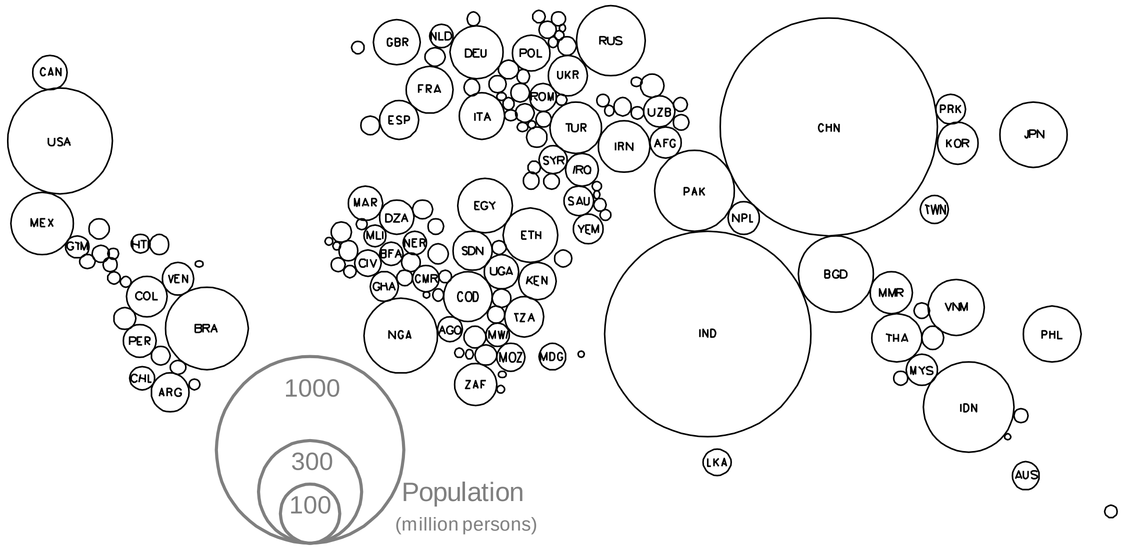


Figure 3. World population in 2010 on circular cartogram.

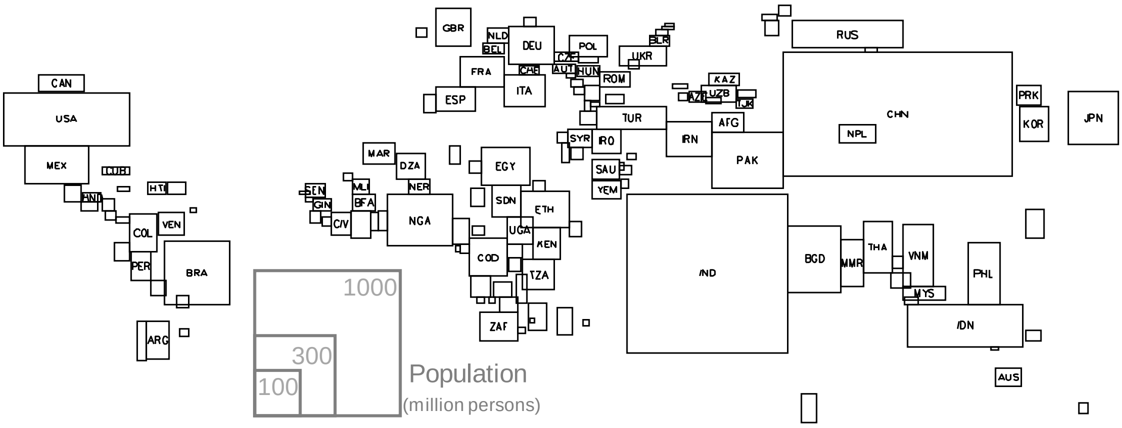


Figure 4. World population in 2010 on rectangular cartogram.

## 5. Conclusions and Future Works

In this study, we have proposed solutions for circular and rectangular cartogram construction by formulating these problems as constrained non-linear optimization problems. The application of the proposed solution revealed that the solution for circular cartogram construction is able to output cartograms without overlaps of circles; however, the solution for rectangular cartogram construction is not and it needs farther improvement. The comparison between the outputs of proposed solutions and that of the previous method and the evaluation of outputs are left for the future works.

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